# A NOTE IN THE SKYRME MODEL WITH HIGHER DERIVATIVE TERMS

Jorge Ananias Neto
Centro Brasileiro de Pesquisas Físicas,
R. Dr. Xavier Sigaud 150 22290-180 Rio de Janeiro, Brazil

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**Abstract**

Another stabilizer term is used in the classical Hamiltonian of the Skyrme Model that permits in a much simple way the generalization of the higher-order terms in the pion derivative field. Improved numerical results are obtained.

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It is well established that the Skyrme Model[1] reproduces with relative success most of the static properties of Nucleon(within approximately $\sim 30\%$). The idea consists of treating baryons as soliton solutions in the Non-Linear Chiral SU(2) Sigma Model whose original Lagrangian is

$$L_1 = \frac{F_\pi^2}{16} \int d^3r \, Tr \left( \partial_\mu U \partial^\mu U^+ \right) , \tag{1}$$

where U is a SU(2) matrix and $F_\pi$ is the pion decay constant. According to Derrick theorem [2] it is necessary to add to expression (1) an adhoc stabilizer term

$$L_2 = \frac{1}{32e^2} \int d^3r \, Tr \left[ U^+ \partial_\mu U, U^+ \partial_\nu U \right]^2 , \tag{2}$$

where $e$ is a dimensionless parameter.

The physical properties are then calculated making use of the semi-classical expansion of the Quantum Hamiltonian where we perform a rotacional collective coordinate expansion[3] $U(r,t) = A(t)U_0(r)A^+(t)$, where A is a SU(2) matrix and $U_0$ a static soliton solution. Adopting the hedgehog ansatz $U = \exp i\tau.\hat{r}F(r)$ where $\tau$ is the Pauli matrix and F(r) is called the chiral angle, the Hamiltonian with the rotational mode can be written as

$$H = M + \frac{l(l+2)}{8I} , \quad l = 1, 2, \ldots , \tag{3}$$

where $M$ is the classical energy and $I$ is the inertia moment[3].

Dubé and Marleau [4] indicate therefore a possible way of generalizing the Skyrme Model introducing higher-order terms in the derivatives of the pion field.In order to simplify the model, they made a particular choice of the Hamiltonian parameters that permited to sum the series in an exponential funcional form. Using this method they obtained improved numerical results.

In the present note we propose to introduce another four derivative stabilizer term [5]given by

$$L_2 = \int d^3r \, c_2 \left[ tr \left( \partial_\mu U \partial^\mu U^+ \right) \right]^2 . \tag{4}$$

As this term is the square of the kinetic term defined in (1), it is natural to take it as a pattern for the inclusion of the higher-order derivative terms. Thus, the standard form of the Lagrangian terms is

$$L_n = \int d^3r \, c_n \left[ Tr \left( \partial_\mu U \partial^\mu U^+ \right) \right]^n , \tag{5}$$

where n=1,2,... . It must be noted that this form is also invariant under chiral transformation (U transforming under $U \rightarrow AUB^{-1}$, A and B are SU(2) matrices), and does not destabilize the soliton solution, since we can to define a positive static Hamiltonian.

To simplify the calculations we will adopt the Sugawara form, $L_\mu = U^+\partial_\mu U$, whose the static component can be written as $L_i = i\tau^a L_i^a$. Using the hedgehog ansatz, the kinetic static term (1) becomes then,

$$L_1 = -c_1 \int d^3r\, Tr\left[\partial_i U \partial_i U^+\right] = c_1 \int d^3r\, tr\left[L_i L_i\right] = -2c_1 \int d^3r\, L_i^a L_i^a\,, \quad (6)$$

where $L_i^a L_i^a = \frac{2\sin^2 F}{r^2} + F'^2$, and $c_1 = F_\pi^2/16$. Since the classical energy is given by -L, where L is the generalized static Lagrangian, we can write the classical total positive energy with the inclusion of the higher order derivative terms as

$$\begin{aligned} E &= \int d^3r \left[c_1\left(2L_i^a L_i^a\right) + c_2\left(2L_i^a L_i^a\right)^2 + \ldots + c_n\left(2L_i^a L_i^a\right)^n\right] \\ &= \int d^3r\ 2L_i^a L_i^a\, c_1 \left[1 + \tfrac{c_2}{c_1}\left(2L_i^a L_i^a\right) + \tfrac{c_3}{c_1}\left(2L_i^a L_i^2\right)^2 + \ldots + \tfrac{c_n}{c_1}\left(2L_i^a L_i^a\right)^{n-1}\right] .(7) \end{aligned}$$

There are many particular values of the relations $\frac{c_n}{c_1}$ in(7) that allow the energy series to summed. Following Dubé and Marleau, if we naively set

$$K_n \equiv \frac{c_n}{c_1} = \frac{1}{(n-1)!}\ \frac{1}{\left(2e^2F_\pi^2\right)^{n-1}}\,, \quad (8)$$

the series of the classical energy(7) converges in an exponential form

$$\begin{aligned} E &= \int d^3r\, 2c_1 L_i^a L_i^a \exp\left[\tfrac{L_i^a L_i^a}{e^2F_\pi^2}\right] \\ &= \tfrac{F_\pi^2}{16}\int d^3r 2\left[\tfrac{2\sin^2 F}{r^2} + F'^2\right]\exp\left[\tfrac{\frac{2sin^2F}{r^2}+F'^2}{e^2F_\pi^2}\right] \\ &= \tfrac{F_\pi}{e}\,\tfrac{\pi}{2}\int_0^\infty dx x^2\left[\tfrac{2sin^2F}{x^2} + F'^2\right]\exp\left[\tfrac{2sin^2F}{x^2} + F'^2\right]\,, \end{aligned} \quad (9)$$

with $F_\pi$ and $e$ being the only input parameters. In expression(9) we have used the series representation $\exp y = \sum_{k=0}^{\infty}\frac{y^k}{K!}$ and a dimensionless variable x defined by

$$x = eF_\pi r\,. \tag{10}$$

It is interesting to point out that with the choice of standard form(5) and with the definition (10), all the coupling constants are absorbed in the new dimensionless variable x. From (9) the variational equation is

$$\begin{aligned}
&\left[2x^2+2x^2S+8x^2F'^2+4x^2SF'^2\right]F''+8\sin 2FF'^2\\
&+4S\sin 2FF'^2+4xF'-\tfrac{16sin^2FF'}{x}+4xSF'\\
&-\tfrac{8Ssin^2FF'}{x}-2sin2F-2Ssin2F=0\,,
\end{aligned}\tag{11}$$

where $S\equiv\left[\frac{2\sin^2F}{x^2}+F'^2\right]$. The soliton solution with the baryon number 1 has the boundary conditions $F(x)=\pi$ at $x=0$ and $F(x)=0$ at $x\rightarrow\infty$. Then, we impose the asymptotic behaviour of F given by(11)

$$\lim_{x\rightarrow\infty}F(x)=\frac{B}{x^2}\,, \tag{12}$$

where B is a constant, to obtain using numerical integration the soliton solution. In figure 1, we show the numerical behaviour of parameter B, which is direclty proportional to the axial vector constant coupling, $g_A=\frac{2\pi}{3}\frac{B}{e^2}$.

The inertia moment is calculated similarly to the series of the classical energy. Performing the rotacional collective coordinate expansion of the classical Lagrangian and picking up only terms linear in $Tr(\partial_0A^+\partial_0A)$ , we obtain the expression of the series of the inertia moment given by

$$I=\int d^3r\frac{8}{3}\sin^2F\;c_1\left[1+2K_2\left(2L_i^aL_i^a\right)+\ldots+nK_n\left(2L_i^aL_i^a\right)^{n-1}\right]\,. \tag{13}$$

With the substitution of $K_n$ by the expression(8), using the dimensionless variable $x$ defined in(10) and taking the limit $n\rightarrow\infty$, we find

$$I = \frac{2\pi}{3} \frac{1}{F_\pi e^3} \int_0^\infty dx x^2 \sin^2 F \left[1 + F'^2 + \frac{2\sin^2 F}{x^2}\right]$$
$$\exp\left[F'^2 + \frac{2\sin^2 F}{x^2}\right] , \tag{14}$$

where we have used the series representation $\exp y(1+y) = \sum_{k=0}^{\infty} \frac{y^k(k+1)}{k!}$ . The numerical calculations are then performed using the masses of the Nucleon ($M_N = 939 Mev$) and of the Delta ($M_\triangle = 1232 Mev$) as input parameters to determine the pion decay constant $F_\pi$ and the dimensionless Skyrme parameter $e$. The main physical results are shown in table 1, according to[3].

Fig 1. Behaviour of the parameter B definided by $B \equiv x^2 F(x)$ , where F(x) is the numerical variational solution of the classical Hamiltonian including terms up to order 4(n=2), 6(n=3), 8(n=4) (solid line) and all orders (dashed line) in derivatives of pion field.

TABLE 1- Physical parameters in the Skyrme Model

| | Adkins[3] | Marleau[4] | This model | Expt. |
|---|---|---|---|---|
| $\mathrm{F}_{\pi}(Mev)$ | 129 | 146 | 144 | 186 |
| $e$ | 5.45 | 8.69 | 6.82 | - |
| $< r^2 >^{\frac{1}{2}}_{I=0}\ (fm)$ | 0.59 | 0.60 | 0.61 | 0.72 |
| $\mu_p$ | 1.87 | 1.89 | 1.90 | 2.79 |
| $\mu_n$ | -1.33 | -1.32 | -1.31 | -1.91 |
| $g_A$ | 0.61 | 0.71 | 0.80 | 1.23 |

Our results indicate an improvement in the physical values of the pion decay constant, $F_\pi$ , and the axial coupling constant, $g_A$. The others values, i.e., the magnetic moments and the isoescalar mean square radius remain basicaly the same obtained by Adkins[5] et al. and Marleau[4]. This procedure, without doubt, improves the physical results. In order to obtain better results for quantities like magnetic moments and the isoescalar mean square radius, we should treat the quantization of the classical Hamiltonian (collective coordinate quantization [3]) in more formal way, using the information about constraint that is present in the system[6]. This particular study will be object of a forthcoming paper [7].

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